Early formation of galaxies initiated by clusters of primordial black holes

V. I. Dokuchaev* and Yu. N. Eroshenko†
Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia

S. G. Rubin‡
Moscow State Engineering Physics Institute, 31 Kashirskoe Sh., Moscow 115409, Russia

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Model of supermassive black holes formation inside the clusters of primordial black holes is developed. Namely, it is supposed, that some mass fraction of the universe $\sim 10^{-3}$ is composed of the compact clusters of primordial (relic) black holes, produced during phase transitions in the early universe. These clusters are the centers of dark matter condensation. We model the formation of protogalaxies with masses about $2\,10^8 M_\odot$ at the redshift $z = 15$. These induced protogalaxies contain central black holes with mass $\sim 10^5 M_\odot$ and look like dwarf spheroidal galaxies with central density spike. The subsequent merging of induced protogalaxies and ordinary dark matter haloes corresponds to the standard hierarchical clustering scenario of large-scale structure formation. The coalescence of primordial black holes results in formation of supermassive black holes in the galactic centers. As a result, the observed correlation between the masses of central black holes and velocity dispersion in the galactic bulges is reproduced.

I. INTRODUCTION

The problem of galaxy formation with supermassive central black hole (BH) becomes more and more intriguing and ambiguous in view of discovery of distant quasars at redshifts $z > 6$ in Sloan Digital Sky Survey [1]. The maximum observed red-shift $z = 6.41$ belongs to the quasar with luminosity corresponding to the accretion onto BH with the mass $3\,10^9 M_\odot$ [2]. Such an early formation of BHs with masses $\sim 10^9 M_\odot$ is a serious difficulty for the standard astrophysical models of supermassive BH formation in galaxies supposing a fast dynamical evolution of the central stellar clusters in the galactic nuclei (see e. .g. [3, 4, 5, 6, 7] and references therein), a gravitational collapse of supermassive stars and massive gaseous disks in galactic centers (see e. .g. [7, 8, 9, 10]), the multiple coalescences of stellar mass BHs in galaxies (see e. .g. [11, 12, 13, 14]) with the subsequent multiple merging of galactic nuclei in collisions of galaxies in clusters (see e. g. [15, 16, 17, 18, 19, 20, 21]). All standard astrophysical (or galactic) scenarios of supermassive BH origin predict a rather late time of supermassive BH formation in the galactic nuclei. An other difficulty is that all these astrophysical scenarios are realized only in strongly evolved galactic nuclei. In view of these problems the cosmological scenarios of massive primordial BHs formation become attractive [22, 23, 24, 25, 26, 27]. In cosmological scenarios the seeds of supermassive BHs are formed long before the formation of galaxies. These primordial black holes (PBH) can be the centers of baryonic [28] and dark matter (DM) [29] condensation into the growing protogalaxies. There are proposed two alternative possibilities: (i) a formation of initially massive primordial BHs and their successive growth up to $\sim 10^9 M_\odot$ due to accretion of ambient matter or (ii) a formation of small-mass primordial BHs and their subsequent merging into the supermassive ones in the process of hierarchical clustering of protogalaxies.

An effective cosmological mechanism of massive primordial BH formation and their clusterization was developed in works [26, 30, 31]. In this papers the properties of spherically symmetric primordial BH clusters were investigated. As a basic example, a scalar field with the tilted Mexican hat potential had been accepted. The properties of resulting primordial BH clusters appear to be strongly dependent on the value of initial phase. In addition, the properties of these clusters depend on the tilt value of the potential Λ and the scale of symmetry breaking f at the beginning of inflation stage. As a result, the mass distribution of primordial BH clusters could vary in a wide range. In our previous paper [32] we considered the model parameters leading initially to large clusters with a rather heavy mass of the central primordial BH, $\sim 4\,10^7 M_\odot$. These central heavy primordial BH can grow due to accretion up to $\sim 10^9 M_\odot$ and, therefore, may explain the observed early quasar activity.

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*Electronic address: dokuchaev@lngs.infn.it
†Electronic address: erosh@inr.npd.ac.ru
‡Electronic address: sergeirubin@list.ru

The elaboration of a discussed mechanism of cosmological primordial BH formation is far from completion and detailed elaboration. For example, there are no now any physical substantiations for reconstruction of scalar field potential parameters and initial characteristics of primordial BH clusters. It is connected not only with the uncertainties of observational data but also with complexities of phase transition details. For example, the domain walls formed during the phase transition in the early universe has a topology of sphere but with a very complicated surface form. When these closed domain walls are turned out inside the horizon, they become self-gravitating. Inside the horizon domain walls tend to obtain a spherical form due to surface tension, but at the same time they strongly oscillate and generate gravitational and scalar waves. As a result their mass gradually diminish. An approximate consideration of this effect [30] demonstrate that for a wide range of initial theoretical free parameters there are conditions for formation of cluster with a supermassive primordial BH. For this reason we will not fix here the definite values of free parameters in the discussed cosmological model of massive black hole formation. The influence of non-sphericity of the formed domain walls see in [30, 31]. An application of the mechanism found in [26, 30, 31] is not limited by a specific form of the scalar field potential. Below we demonstrate that substantial number of potentials, like e. g. those used in hybrid inflation, also result in formation of massive BHs. Moreover, it is hard to avoid primordial BHs overproduction in the early Universe. In fact, any inflationary model using potential with two or more minima must take into account this mechanism of primordial BHs overproduction.

In this paper we choose parameters of the potential which lead to formation of relatively small primordial BH clusters. We suppose that relatively small primordial BH clusters provide the major contribution to initial density perturbations which afterwards evolve into protogalaxies. The hierarchical clustering of protogalaxies during the cosmological time leads to the observable large scale structure. We describe the gravitational dynamics of DM coupled with primordial BH clusters and demonstrate that a protogalaxy could be formed without any initial fluctuations in DM density. In this case the clusters of primordial BHs play the role of initial fluctuations. Two scenarios of supermassive BHs formation could coexist: (i) the most massive clusters of primordial BHs account for an early quasar activity [32], but (ii) less massive (considered in this paper) clusters of primordial BHs produce more numerous supermassive BHs observed nowadays in almost all structured galaxies.

There are several stages of BHs and galaxies formation in the described scenario: (i) Formation of closed walls of scalar field just after the end of inflation with a subsequent collapse some of these walls with formation of massive primordial BH cluster with the most massive BH in the center after the horizon crossing according to [26, 30]. (ii) Detachment of the central dense region of the primordial BH cluster from cosmological expansion and virialization. Numerous small-mass BHs merge with a central one. (iii) Detachment of the outer cluster region (where DM particles dominate) from cosmological expansion and a protogalaxy growth. Termination of a protogalaxy growth due to interaction with the surrounding standard DM fluctuations. (iv) Gas cooling and star formation accompanied by the merging of protogalaxies and final formation of modern galaxies.

II. FORMATION OF PRIMORDIAL BLACK HOLES IN HYBRID INFLATION

It is instructive to consider the mechanism of massive primordial BH production in the framework of the hybrid inflationary model [33] following to results of paper [34]. According to [33] the hybrid inflation potential has the form

$$V(\chi,\sigma) = \varkappa^2\left(M^2 - \frac{\chi^2}{4}\right)^2 + \frac{\lambda^2}{4}\chi^2\sigma^2 + \frac{1}{2}m^2\sigma^2. \tag{1}$$

Inflation proceeds during a slow rolling along the valley $\chi = 0, \sigma > \sigma_c$. When the field σ decreases up to $\sigma_c = \sqrt{2}\frac{\varkappa}{\lambda}M$, the motion along the line $\chi = 0$, $\sigma < \sigma_c$ becomes unstable, and the field χ quickly moves to one of the minima $\chi_\pm = \pm 2M, \sigma = 0$, see Fig. 1. Inflation is finished producing strong fluctuations around the accidentally chosen minimum. This rather well elaborated picture suffers a serious problem nevertheless. During an inflationary stage, when the fields σ and χ move classically along the line $\chi = 0$, the space is divided in many causally disconnected space regions due to quantum fluctuations. The values of scalar fields in neighboring regions (or domains) are slightly different. Each e-fold time interval produces approximately $e^3 \simeq 20$ different regions. Hence there are about $e^{180} \simeq 10^{78}$ space domains right before the end of inflation. The values of field in different domains chaotically distributed around the point $\chi = 0, \sigma = \sigma_c$. The domains with the field value $\chi < 0$ tend to the left minimum $\chi_- = -2M, \sigma = 0$. Another part go to the right minimum $\chi_+ = +2M, \sigma = 0$. A lot of walls between such domains appear and we run into the well known problem of wall-dominated Universe [35].

The only way for our Universe to evolve into recent state is to be created with a nonzero initial field value, $\chi_{\rm in} \neq 0$ at the beginning of inflation. During inflation, a nonzero field χ is slowly approaching the critical line $\chi = 0$. If, in the middle of inflation, a field with average value approaches to the critical line $\chi = 0$, the fluctuations of the field in some part of space domains could cross this line. In future, these domains will be in a vacuum state, say, χ_- surrounded

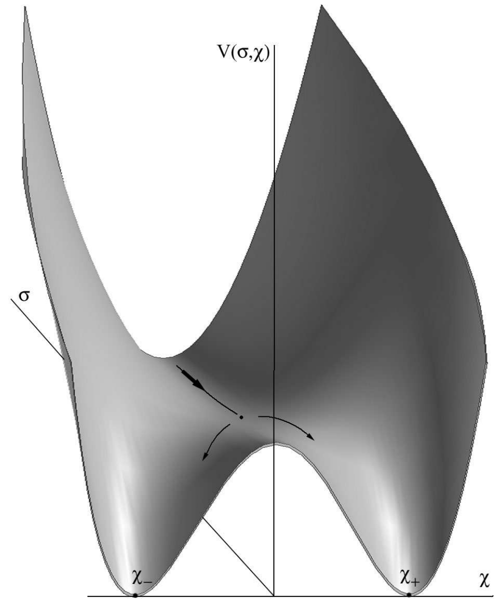

FIG. 1: Potential for the hybrid inflation model. Arrows show the directions of classical motion of the evolving scalar field.

by a sea of another vacuum χ_+. The two vacua domains are separated by a closed wall as it was discussed above. A number of these walls strictly depends on the initial conditions at the moment of our Universe creation, i. e. at the beginning of inflation.

Let us estimate the energy and size of the formed closed walls by supposing that a field in some space domain crosses the critical line during the time corresponding to number N of e-folds before the end of inflation. A characteristic size of this domain is of the order of the Hubble radius, H^{-1}, and it will increase correspondingly in e^N times up to the end of inflation. A surface energy density of the domain wall after inflation for potential 1 is

$$\epsilon = \frac{8\sqrt{2}}{3}\varkappa M^3. \tag{2}$$

A resulting total energy $E_{\rm wall}$ of the wall after inflation is approximately

$$E_{\rm wall} \simeq 4\pi\epsilon \left(H^{-1}e^N\right)^2 = 4\sqrt{2}\frac{M_{\rm Pl}^2}{\varkappa M}e^{2N}, \tag{3}$$

where a numerical value of N is in interval $(0 < N < N_U \simeq 60)$. These walls collapse into BH with mass $M_{\rm BH} \simeq E_{\rm wall}$ (see [30] for details). Let us estimate the mass-scale of these BHs $M_{\rm BH}$ for the characteristic values of parameters $\varkappa = 10^{-2}$ and $M = 10^{16}$ GeV. For $N = 40$ we obtain $M_{\rm BH} \simeq 3\,10^{59}$ GeV $\sim 100{\rm M}_\odot$. The same estimation of the minimum mass of BHs created at the e-fold number $N = 1$ before the end of inflation gives $M_{\rm BH,min} \simeq 10^6 M_{\rm Pl}$. As a result the hybrid inflation leads to BH production with mass in the wide range 10^{25} GeV $< M_{\rm BH} < 10^2{\rm M}_\odot$. An abundance of massive BHs depends on the proximity of an average field value to critical the line $\chi = 0$which in turn, depends on the initial conditions and specific values of model parameters.

The main finding of this consideration is that passive primordial BH caused by phase transition during inflation is the rule rather than exception. In this paper we elaborate the idea that clusters of primordial BHs could be the seeds for galaxy formation. This work is based on the results of [26, 30, 31], where the "Mexican hat" potential was considered. Calculations based on this potential provide the suitable framework for consideration of protogalaxies formation around the clusters of primordial BHs.

An initial modeled mass profile $M_h(r_i)$ of primordial BH in the cluster (see details in [36]) is shown in the Fig. 2. This numerically calculated profile is a starting point for study made below. For comparison in the Fig. 2 is shown also a radial distribution of DM mass $M_{\rm DM}(r_i)$ inside the same sphere. Radius r_i is a size of sphere at the moment t_i and the temperature T_i, when this sphere is crossing the cosmological horizon.

Note that different spheres in the Fig. 2 are shown at different times t_i. Due to cosmological expansion the radial mass distribution of uniform DM does not follow the law $M_{\rm DM} \propto r^3$ as it must be for fixed time. A physical size of chosen sphere at temperature T_i is smaller than one in the recent epoch in T_0/T_i times, where $T_0 = 2.7$ K. A total mass in the central parts of the BH cluster is so high, that some part of BHs appear to be inside the combined

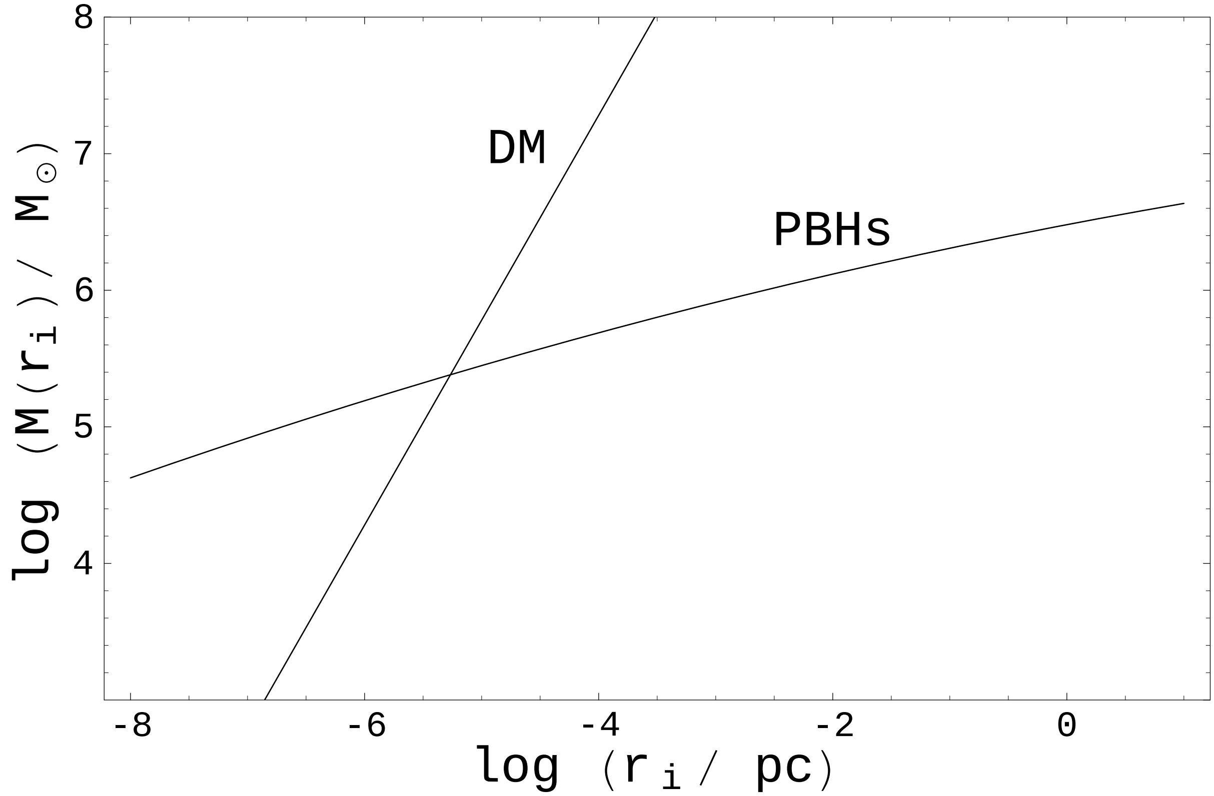

FIG. 2: The initial mass profiles of primordial BHs $M_h(r_i)$ and DM $M_{\rm DM}(r_i)$ in the cluster (protogalaxy).

gravitational radius $r_g = 2GM/c^2$. An initial total mass of BHs inside combined event horizon is $2.7\,10^4 M_\odot$. A massive BH of this mass becomes the most massive central BH in the cluster.

III. GRAVITATIONAL DYNAMICS OF BH CLUSTER AND DARK MATTER

Let us describe a gravitational dynamics of the primordial BH cluster and internal DM in the combined gravitational field. In a central part of the cluster the primordial BHs dominate in mass. This central part of the cluster is detached from cosmological expansion at the radiation dominated epoch. Conversely, the DM dominates at the outer part of the cluster, and the outer part is detached from cosmological expansion much more later at the matter dominated epoch.

Consider a spherically symmetric system with radius $r < ct$, consisting of (i) primordial BHs with a total mass M_h inside the radius r, (ii) radiation with energy density ρ_r, (iii) DM with density $\rho_{\rm DM}$ and (iv) vacuum with an energy density ρ_Λ. The radiation density (and obviously the density of vacuum) is homogenous. The corresponding fluctuations induced by primordial BHs are classified as entropy fluctuations. Because the characteristic radial scale is much more less then the instant horizon scale, we use in a standard way the Newtonian gravity but will take into account the prescription of [37] to treat the gravity of homogenous relativistic components $\rho \to \rho + 3p/c^2$. The dynamical evolution of spherical shell with an initial radius r_i obeys the equation

$$\frac{d^2r}{dt^2} = -\frac{G(M_h + M_{\rm DM})}{r^2} - \frac{8\pi G\rho_r r}{3} + \frac{8\pi G\rho_\Lambda r}{3}, \qquad (4)$$

with an initial conditions at time t_i: $\dot{r} = Hr$ and $r(t_i) = r_i$. Equation (4) is derived by taken into account that $\varepsilon_r + 3p_r = 2\varepsilon_r$ and $\varepsilon_\Lambda + 3p_\Lambda = -2\varepsilon_\Lambda$. With these initial conditions the shell initially is growing in radius but expansion decelerates with time according to (4). At some time the expansion is stopped, the shell separates from cosmological expansion and start to shrink. All types of constituent nonrelativistic internal matter in the cluster — the dark matter, primordial BHs and baryons follow the shell dynamics. As a result, the solutions of (4) for shells with different initial radii supply us with density distribution of the dark matter and PBH. For numerical calculations

it is useful to rewrite equation (4) by using dimensionless variables:

$$r(t) = \xi a(t) b(t), \tag{5}$$

where ξ is a comoving length, $a(t)$ is a dimensionless scale factor of the universe normalized to the present moment t_0 as $a(t_0) = 1$ and dimensionless function $b(t)$ describes the deflection of a chosen shell from the cosmological expansion (from the Hubble law). A comoving length ξ is related with a total mass of DM inside considered spherical volume (i. e. excluding total BH mass) by the relation $M_{\rm DM} = (4\pi/3)\rho_{\rm DM}(t_0)\xi^3$, where $\rho_{\rm DM}(t_0)$ is the nowadays DM density. A scale factor $a(t)$ obeys one of the Friedman equation, which can be rewritten as $\dot{a}/a = H_0 E(z)$, where redshift $z = a^{-1} - 1$, H_0 is a present value of the Hubble constant and function

$$E(z) = [\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\Lambda,0}]^{1/2}, \tag{6}$$

where $\Omega_{r,0}$ is the present density parameter of radiation, $\Omega_{m,0} \simeq 0.3$, $\Omega_{\Lambda,0} \simeq 0.7$, and $h = 0.7$. By using the Friedman equation for $\ddot{a}$ one can rewrite an evolution equation (4) as follows

$$\frac{d^2 b}{dz^2} + \frac{db}{dz} S(z) + \left(\frac{1+\delta_h}{b^2} - b \right) \frac{\Omega_{m,0}(1+z)}{2E^2(z)} = 0, \tag{7}$$

and $\delta_h = M_h / M_{\rm DM}$ is a fluctuation amplitude and function

$$S(z) = \frac{1}{E(z)} \frac{dE(z)}{dz} - \frac{1}{1+z}. \tag{8}$$

In the limiting case $\Omega_\Lambda = 0$ equation (7) is equivalent to equation obtained in [38]. We start to trace out the evolution of primordial BH cluster starting from an initial high redshift z_i, when the considered shell crosses the cosmological horizon $r \sim ct_i$. The initial conditions for this problem are shown in the Fig. II.

The most early epoch in our calculation corresponds to formation of the central most massive BH with a mass $2.7\,10^4 M_\odot$, described at the end of preceding Section. A corresponding temperature of the universe at that time is $T \simeq 16$ MeV. In cosmological scenario with the standard perturbation spectrum a mass of primordial BH could not exceed a total mass under the instant horizon $M \sim (t/t_{\rm Pl}) M_{\rm Pl}$ [39]. As a result, at temperature $T \simeq 16$ MeV the mass of primordial BH cannot be larger than $10^3 M_\odot$. Nevertheless, in the considered scenario the mass of primordial BHs is much more larger because they are formed from the collapsing domain walls, not from initial fluctuations. At the same time a total energy of domain walls could be rather large because they are formed and stretched during inflation. We suppose also that DM has been already decoupled from radiation at this temperature. For example, in the case neutralino DM particles with mass 100 GeV and slepton mass 1 TeV a kinetic decoupling temperature is $\simeq 150$ MeV [40], corresponding to a much more earlier epoch. Therefore, the neutralino DM particles at the time of primordial BH cluster formation are influenced only by gravitational forces and the combine clustering of two-component medium (BHs+DM) is described by a single equation (7) from the very beginning. The same situation is realized for DM composed of super-heavy particles with mass $m_\chi \sim 10^{13} - 10^{14}$ GeV which probably never been in kinetic equilibrium with radiation. In the opposite case (for some other DM particles candidates) the growth of fluctuations in DM medium is suppressed by friction due to interaction with radiation while BHs are clustering. The super-heavy particles are more preferable for our model in comparison with the neutralinos because their annihilation cross section is very small $\propto m_\chi^{-2}$. In this case the are no problems with a possible huge annihilation rate in the central part of considered cluster.

The amplitude of fluctuations produced by primordial BH cluster $\delta^h_{\rm eq}(M_{\rm DM}) = 2.5\delta_i(M_{\rm DM})$ is shown in the Fig. 3. The numerical factor 2.5 corresponds to entropy perturbations which grow up to time $t_{\rm eq}$ according to the Meszaros solution [41]. In the Fig. 3 the r.m.s. values of standard inflation fluctuations are shown. For standard DM fluctuations we use the fitting formula for power spectrum from [42]:

$$P(k) = \frac{Ak}{(1 + 1.71u + 9u^{1.5} + u^2)^2}, \tag{9}$$

where $u = k/[(\Omega_{m,0} + \Omega_{b,0})h^2\ {\rm Mpc}^{-1}]$, and k is a comoving wave vector in ${\rm Mpc}^{-1}$ units. The initial spectrum is supposed to be the Harrison–Zeldovich type. The relation between the mass scale M of the r.m.s. perturbations and the linear-scale R is

$$\sigma(M) = \frac{1}{2\pi^2} \int\limits_0^\infty k^2\, dk\, P(k) W(k,R), \tag{10}$$

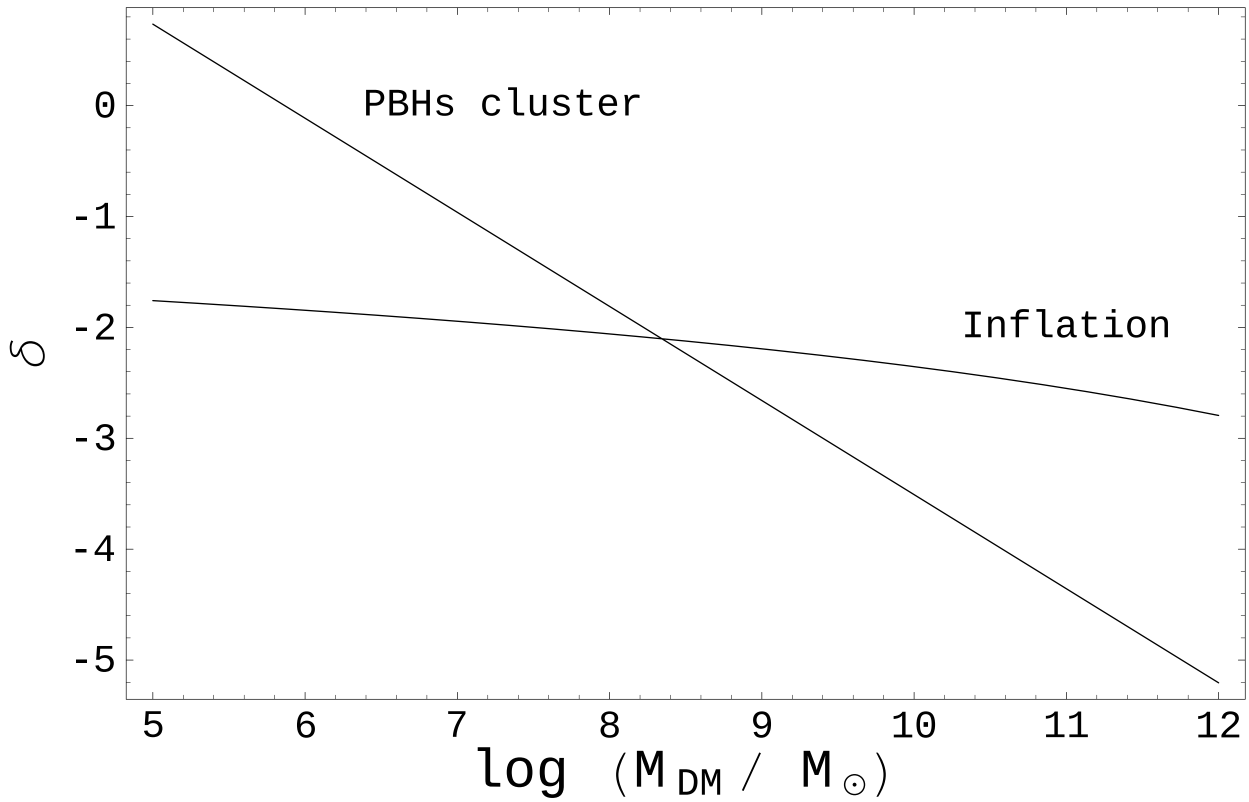

FIG. 3: The r.m.s. density perturbations at the time $t_{\rm eq}$ of matter-radiation equality are presented. Two cases are compared — a total density fluctuations produced in the presence of primordial BH clusters and the standard ones generated by inflation.

where $W(k, R)$ is a filtering function [43]. We put for estimations $\delta_{\rm eq} \simeq \sigma_{\rm eq}$. The normalization constant A in (9) corresponds to the observable value 0.9 of r.m.s. fluctuations at 8 Mpc scale at the recent time.

The termination of shell expansion, $\dot{r} = 0$, at the instant of time t_s corresponds to condition $db/dz = b/(1+z)$ in accordance with the definition of function b. Following to [41] we suppose that every chosen shell after the termination of expansion is virialized and contracted from the maximum radius $r_s = r(t_s)$ to the radius $r_c = r_s/2$. The resulting average density of DM in the virialized shell ρ is 8 times larger than one at the time of maximum shell expansion:

$$\rho = 8\rho_{m,0}(1+z_s)^3 b_s^{-3}, \tag{11}$$

where $b_s = b(t_s)$ and an effective (virialized) shell radius is

$$r_c = \left(\frac{3}{4\pi}\frac{M_{\rm DM}}{\rho}\right)^{1/3}. \tag{12}$$

Numerical solution of equation (7) is shown in the Fig. 4 and represents the growth of protogalaxy radius with time (or redshift z) in the absence of standard DM fluctuations. This numerical solution is valid up to the time when DM fluctuations start to grow effectively.

Let us trace a time evolution of the described spherical cluster (protogalaxy) step by step starting from its central region. It is obvious (and also validated by our numerical solution) that expansion of more dense inner spherical shells stops earlier than a corresponding expansion of rarefied outer shells. A central most massive BH with mass $M_c = 2.7\,10^4 M_\odot$ forms in the the cluster of at the very early time as discussed in the Sec. II. The dense central spherical shells are detached from the cosmological expansion very early, at the radiation dominated epoch. In in later time, nearly all matter of these central shells will be accreted by the central BH in the process of two-body relaxation of BHs. We will describe this process below.

The process similar to "secondary accretion" (i. e. a gravitational contraction of initially homogeneous DM around a central mass) takes place for the early formed primordial BHs. As a result, the cluster of primordial BHs would be "enveloped" by an extended DM halo. We call these haloes the "induced galaxies" (IG). The resulting density profile in the cluster does not follow the secondary accretion law $\rho \propto r^{-9/4}$ [41] because a central mass in our case is

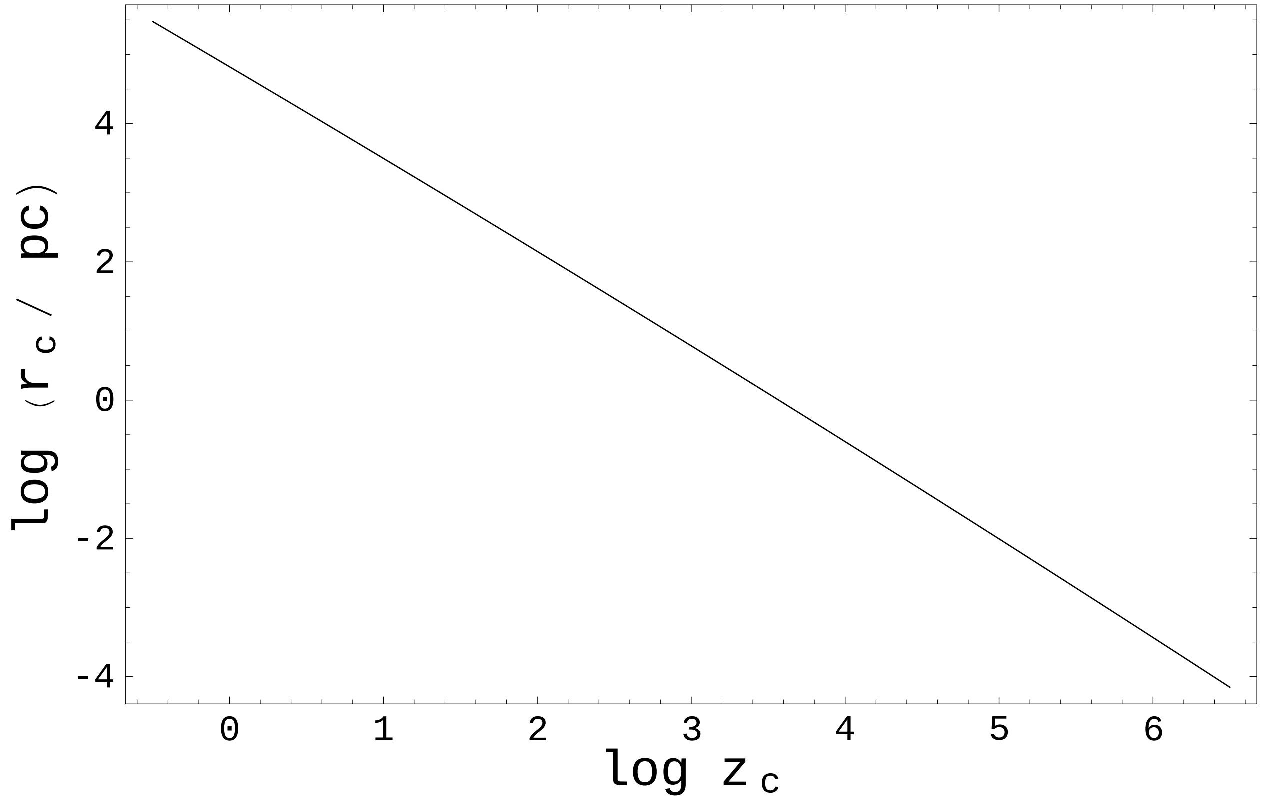

FIG. 4: The virial radius of protogalaxy r_c is shown as a function of redshift z.

noncompact. The distribution of DM in the cluster after the virialization is

$$\rho_{\rm DM}(r) = \frac{1}{4\pi r_c^2}\frac{dM_{\rm DM}(r_c)}{dr_c}\bigg|_{r_c=r}, \tag{13}$$

where function $M_{\rm DM}(r_c)$ is determined from the solution of equation (7), and ρ and r_c from the solution of (11) and (12) respectively.

By analogy with a DM profile (13) one can obtain the corresponding profile for primordial BHs density $\rho_{\rm BH}(r)$ and for the total density $\rho_{\rm DM}(r)+\rho_{\rm BH}(r)$. The results are shown in the Fig. 5, where density is expressed in units $M_\odot/pc^3$ and radial distance is in parsecs. These numerically calculated density profiles with a rather good accuracy are fitted by power laws:

$$\rho_{\rm DM}(r) \;=\; 2.3\,10^4\left(\frac{r}{1\ \rm pc}\right)^{-2.13}\ {\rm M_\odot\ pc^{-3}}, \tag{14}$$

$$\rho_{\rm BH}(r) \;=\; 2.9\,10^3\left(\frac{r}{1\ \rm pc}\right)^{-2.85}\ {\rm M_\odot\ pc^{-3}}. \tag{15}$$

At radial distance $r > 0.056$ pc the local density of DM prevail over the density of BHs, while a total internal mass of DM prevails over a total mass of BH at distance $r > 0.7$ pc. Therefore, the influence of BHs on a subsequent protogalaxy dynamics is limited by the central parsec. This influence will be considered in the next Section. The derived density profile (14) differs from the Navarro-Frenk-White or Moore et al. profiles, obtained in the numerical simulations of DM halo formation but is very near to these profiles at intermediate scales, when power-law index is $\simeq -2$. An interesting properties of the derived density distribution is a diminishing of mean virial velocity $V_v = (GM/2R)^{1/2}$ of IGs with time (or with decreasing of z). This behavior is a consequence of the specific shape of perturbation spectrum produced by clusters of primordial BHs.

A total mass of IG is growing with time because increasingly more distant regions are separated from cosmological expansion and virialized around the central most massive BH. The growth of IG is terminated at the epoch of a nonlinear growth of ambient standard density fluctuations with a mass of the order of IG. These fluctuations

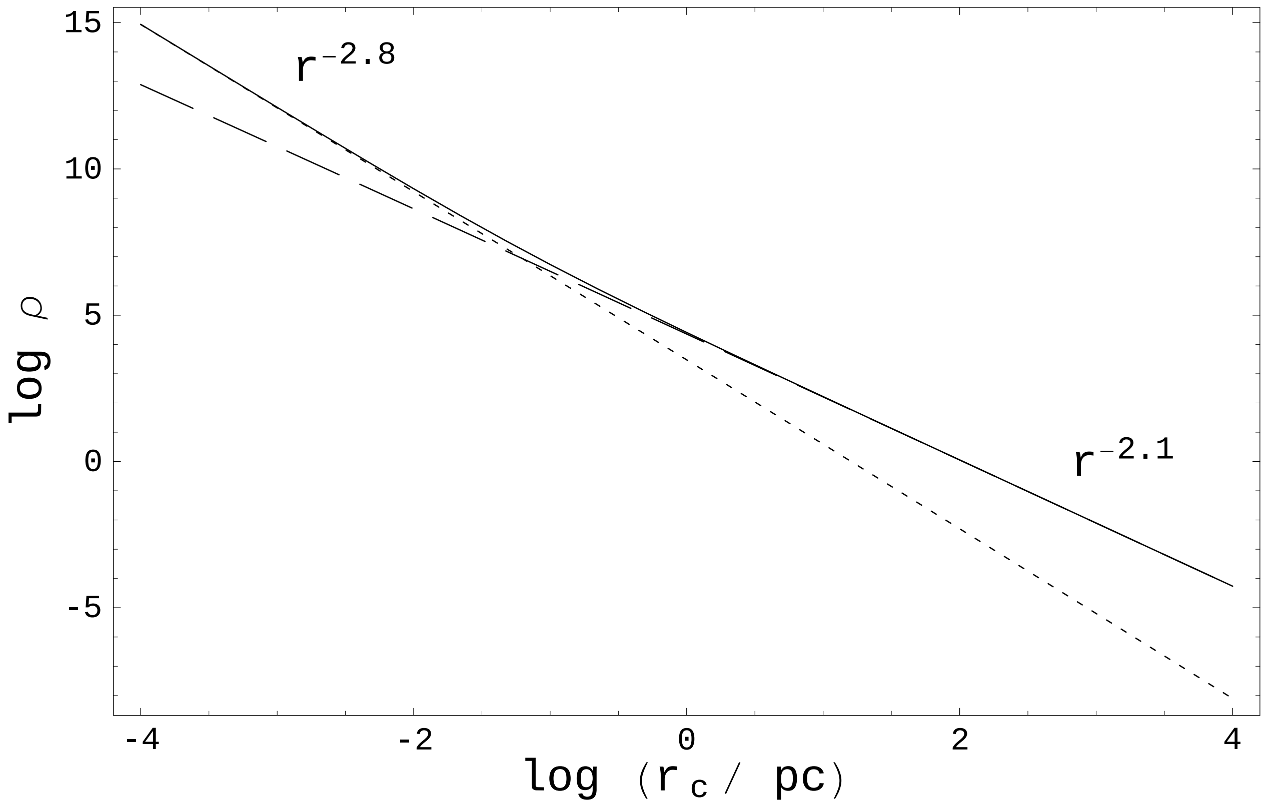

FIG. 5: The final density profile of protogalaxy $\rho(r_c)$ in units $M_\odot/\mathrm{pc}^3$ in dependence of radial distance r_c from the cluster center for DM (dashed line), for BHs (dotted line) and for the sum of DM+BHs (solid line) respectively.

are originated in a standard way from inflation cosmological perturbation spectrum $P(k)$ (see e. g. [43, 44]. The fluctuations of both types are growing in a similar way at the matter dominated epoch. Therefore, a corresponding condition for termination of growth of typical IG due to gravitational instability is

$$\nu\sigma_{\rm eq}(M_{\rm DM}) = \delta^h_{\rm eq}(M_{\rm DM}), \tag{16}$$

where ν is the perturbations peak height (and we consider only a mean perturbation with $\nu = 1$). The r.h.s of (16) is the of fluctuation caused by BH cluster. Respectively the l.h.s. of (16) is the standard Gaussian fluctuations according to (10). Both types of fluctuations are taken at the moment of matter-radiation equality $t_{\rm eq}$. Numerical solution of (16), which is the intersection point of two curves in the Fig. 3, gives the final mass of IG $M_{\rm DM} = 2.2\,10^8 M_\odot$.

The evolution of cosmological perturbations in the Universe with the Λ–term at the matter dominated epoch can be derived from equation (7) or from the corresponding equation of paper [44]:

$$\frac{\delta(t)}{\delta(z_{\rm eq})} = \frac{g(z)}{g(t_{\rm eq})}\frac{(1+z_{\rm eq})}{(1+z)}, \quad g(z) \simeq \frac{5}{2}\frac{\Omega_m}{\Omega_m^{4/7} - \Omega_\Lambda + (1+\Omega_m/2)(1+\Omega_\Lambda/70)}, \tag{17}$$

where $\Omega_m = \Omega_{m,0}(1+z)^3/E^2(z)$, $E(z)$ is from (6) and Ω_Λ is defined in a similar way. Now by fixing the perturbation amplitude $\delta(z_{\rm eq}) = \sigma_{\rm eq}(M_{\rm DM})$ at $M_{\rm DM} = 2.2\,10^8 \mathrm{M}_\odot$, one can find from (17) the instant of growth termination. This happens near the redshift $z = 15$, and so only in the narrow range of mass and radius shown in the Figs. 2 and 4.

A resultant structure of IG is the following: The central BH with mass $2.7\,10^4 M_\odot$ is surrounded by the cluster of smaller BHs with a total mass $2.2\,10^5 M_\odot$ and with a radius $r \sim 0.7$ pc. Outside this sphere the DM prevail in mass and has a density profile (14). A total mass of mean IG (i. e. for $\nu = 1$) is $2.2\,10^8 M_\odot$ and the corresponding virial radius of IG is $R = 1.8$ kpc. In the inner parsec of the IG the two-body relaxation and accretion processes operated from the very outset. Starting from $z = 15$ the IG participates in the hierarchical clustering and a subsequent structure formation proceeds by the standard scenario: small galaxies including IGs are assembled into the larger galaxies, clusters and superclusters. Being formed, the IG looks like a dwarf spheroidal galaxy with a central massive BH surrounded by BHs of intermediate mass and with a central DM density spike shown in the Fig. 5. Some part of these IGs could escape galactic merging and survive to our time.

IV. ACCRETION IN THE INDUCED PROTOGALAXY

Let us describe the accretion of DM and primordial BHs onto the central BH in the described IG (or protogalaxy). The cluster of primordial BHs is composed of BHs with different masses. Therefore, an important factor of dynamical evolution is mass segregation, i. e. a concentration of more massive BHs closer to the center. This mass segregation considerably complicates the treatment of cluster dynamical evolution. We use here an approximate approach by considering the BHs of different masses as independent homological subsystems evolving in the combined gravitational field. This approach is a similar one to used in studies of evolution of multicomponent star clusters.

A total rate of DM accretion onto primordial BHs moving within a radial distance r from the IG center is

$$\dot{M}_{\rm DM} = \sum_i N_i \sigma_{{\rm acc},i} v \rho_{\rm DM}, \tag{18}$$

where N_i is a total number of BHs with mass M_i inside radius r (in reality the mass distribution of BHs is continuous), $v \simeq (GM_{\rm tot}/r)^{1/2}$ is a mean (virial) velocity, $M_{\rm tot} = M_{\rm DM} + \sum M_i N_i$, and a cross-section of DM particle capture by BH is $\sigma_{{\rm acc},i} = \pi r_{g,i}(c/v)^2$, $r_{g,i} = 2GM_i/c^2$. A corresponding "inverse" characteristic accretion time of DM is

$$t_{\rm acc}^{-1} \simeq \frac{\dot{M}_{\rm DM}}{M_{\rm DM}} = \frac{3G^2}{cvr^3} \sum_i N_i M_i^2. \tag{19}$$

By using results of numerical solutions of the preceding Section we find that accretion time of DM (19) is a rather well approximated by the power-law:

$$t_{\rm acc}(r) \simeq 8\,10^3 \left(\frac{r}{1\ {\rm pc}}\right)^{2.7} t_0, \tag{20}$$

where t_0 is age of the Universe. From this relation it follows that DM is totally accreted by now, $t_{\rm acc} \sim t_0$, inside the radius $R_c \simeq 0.036$ pc. Therefore, the DM density profile (14) is valid only at $r \geq R_c$. A total accreted mass of DM is negligible in comparison with a total mass of BHs in the cluster, and so the DM accretion is unimportant for the growth of central BH.

A two-body relaxation time in the cluster of equal mass BHs is [45]:

$$t_{\rm rel} \simeq \frac{1}{4\pi} \frac{v^3}{G^2 m^2 n \ln(0.4N)}, \tag{21}$$

where N, n and m are respectively a total number, number density and individual mass of BHs in the cluster. We will use now for forthcoming estimations only mean values. A characteristic time-life of BH cluster (due to "evaporation" of fast BHs) is $t_e \simeq 40 t_{\rm rel}$ [45]. At the end of this time the gravitational collapse happens, starting with avalanche contraction of the remaining BH cluster. The collapse proceeds shell by shell of starting from the innermost shells. We estimate the corresponding mean values of t_e for shells with different radii by using relation $t(z) = t_e$, where $t(z)$ is age of the Universe corresponding to redshift z. As a result due to this dynamical evolution process the collapsing shells of relaxed BHs increase the mass of central BH.

As well as accretion of DM, the accretion of collapsing shells of primordial BHs provides a rather small contribution to the growth of central BH in the IG.

Indeed, a mass of central BH at $z = 15$, when a growth of IG is terminated, is $M_{\rm BH} = 6.9\,10^4 M_\odot$. This mass is the sum of initial central BH mass $2.7\,10^4 M_\odot$ and a total mass of collapsed shells of primordial BHs at that epoch. At the time of large galaxy formation, corresponding to $z \simeq 1.7$, the central BH mass is $M_{\rm BH} = 7.2\,10^4 M_\odot$. If some IG survived up to nowadays epoch, $z = 0$, it will have now the central BH with mass $M_{\rm BH} = 7.3\,10^4 M_\odot$ (due to accretion of DM and primordial BHs). From these estimations we conclude that a main contribution to the growth of central supermassive BHs in galaxies was provided not by the DM and primordial BH accretion but accretion of baryonic matter (gas and/or stars) and/or merging of galaxies.

V. MERGING OF PROTOGALAXIES AND BLACK HOLES

In Section III we calculated a characteristic mass of IG or protogalaxy $2.2\,10^8 M_\odot$ which is formed around a seed primordial BH cluster at $z = 15$. At this epoch in the vicinity of considered IG there are a lot of smaller neighboring protogalaxies, both ordinary and IG. All these protogalaxies will hierarchically merge later into the large modern galaxies.

The individual IGs are massive enough to sink down by dynamical friction into the galactic center during the Hubble time. The mass loss of IGs due to tidal stripping in spiralling down to the galactic is ineffective due to their large density. Indeed, a condition for tidal stripping of particles at distance r_s from IG center and at distance r from the host galaxy center is the equality of acceleration produced by IG and the tidal acceleration:

$$\frac{GM(r_s)}{r_s^2} = r_s \frac{d}{dr} \frac{GM_{\rm H}(r)}{r^2}, \tag{22}$$

where $M(r_s)$ and $M_{\rm H}(r)$ are respectively the mass profile of IG and the host galaxy. According to the Navarro-Frenk-White model the DM distribution in the Galactic halo is

$$\rho_{\rm H}(r) = \frac{\rho_0}{(r/L)\,(1+r/L)^2}, \tag{23}$$

where $L = 28$ kpc, $\rho_0 = 5\,10^6 M_\odot{\rm kpc}^{-3}$ and for normalization it used the local density in the Sun vicinity. Using the density profile (14) we find from (22) that at any radial distance r the radius of tidal stripping r_s is greater than IG radius 1.8 kpc. Therefore, the IGs sink down to the galactic center as a whole without tidal stripping.

The possibility of IG spiralling down to the galactic center by the influence of dynamical friction depends on the initial orbit of IG. Suppose for estimation that orbit of IG is circular. By using the known expression for dynamical friction force [46] and an equation for the angular momentum loss, one finds the differential equation for orbital radius evolution

$$\frac{dr}{dt} = -\frac{4\pi G^2 M_s(r)\rho_{\rm H}(r)\Lambda B r}{v(r)^3}, \tag{24}$$

where $v(r) = \sqrt{GM_H(r)/r}$, $\Lambda \simeq 10$, $B \simeq 0.427$. As it was shown above, the tidal stripping is ineffective and so the mass of IG $M_s(r) = const = 2.2\,10^8 M_\odot$. Consider at first the density profile (23) for our Galaxy. It was formed when age of the Universe was approximately one fourth of the nowadays days age. Numerical solution of equation (24) demonstrates that only IGs inside the radius 26 kpc have enough time to sink down into the Galactic center. The typical elliptical galaxies are formed earlier than our Galaxy and being much more denser. Therefore, all IGs in elliptical galaxies sank down to their centers.

According to observations the masses of central supermassive BHs in Sa, Sb, Sc galaxies are in general smaller than those in E and S0 galaxies. In our model this is connected with a relatively late formation of Sa, Sb, Sc galaxies, when the main part of primordial BHs have not enough time to sink down to the galactic center. In particular, $\sim 10^3$ BHs with mass $\sim 10^5 M_\odot$ enveloped by IG can inhabit in our Galaxy. They could be observed as the widely discussed ultra-luminous X-ray sources.

The fate of primordial BHs inside the central parsec of the host galaxy is rather uncertain. We suppose that during the Hubble time major part of these BHs are merged into a single supermassive BH. Namely, the dynamical friction is a very effective mechanism for BH merging at final stage because the density of IG, $\rho \propto r^{-2.8}$ (see in the Fig. 5), is strongly growing towards the center and smoothed out only at very small distance $R_c \simeq 0.036$ pc from the central BH. An additional dynamical force is produced by interactions of IGs with stars from the bulge and central star cluster. As a result, the late phase of BHs merging proceeds very fast. The probability of simultaneous presence in the galactic nucleus of three or more BHs is very low due to the slingshot effect. On the contrary, a substantial amount of massive BHs may inhabit the galactic halo, if they turn out rather far from the galactic center from the very beginning [26, 30, 31]. Our assumption of multiple merging of primordial BHs may be violated in the less dense galaxies of late Hubble types.

Multiple coalescence of massive primordial BHs in the galaxies is inevitably accompanied by the strong burst of gravitational radiation. The future interferometric detector LISA is capable to detect these coalescence events. A simple estimation of the burst rate from an observable part of the Universe gives

$$\dot{N}_{grav} \sim \frac{4\pi}{3}\frac{N}{t_0}(ct_0)^3\; n_g \sim 100\left(\frac{n_g}{10^{-2}{\rm Mpc}^{-3}}\right)\left(\frac{t_0}{10^{10}{\rm yrs}}\right)^2\left(\frac{N}{10^3}\right)\;{\rm yrs}^{-1}, \tag{25}$$

where n_g is a mean number density of structured galaxies (galaxies with nuclei) and N is a mean number of merging events per galaxy. Gravitational bursts provide the principal possibility for the verification of considered model by the LISA detector.

VI. CORRELATIONS OF CENTRAL BLACK HOLES WITH BULGES

Recent observations (see e. g. [47]) reveal the correlations between the mass of the central Supermassive BH (SBH) in the galactic nucleus $M_{\rm SBH}$ and velocity dispersion σ_e at the bulge half-optical-radius:

$$M_{\rm BH} = 1.2(\pm 0.2)\, 10^8 \left(\frac{\sigma_e}{200 \text{ km/s}}\right)^{3.75(\pm 0.3)} \text{M}_\odot. \tag{26}$$

In other set of observations [48] a different form of correlation was derived: $M_{\rm SBH} \propto \sigma_e^{4.8(\pm 0.5)}$. We show below that our model of IGs reproduces correlations (26). At the early stage of hierarchical clustering of small protogalaxies into the bigger ones the discussed primordial black holes are homogenously mixed with DM at the scales greater than IGs. For this reason a total mass of these primordial BHs in any galaxy $\sum M_{\rm BH}$ would be proportional to the galactic DM halo mass M. After the final merging of primordial BHs into a single central BH the similar relation retains: $M_{\rm SBH} \propto \sum M_{\rm BH} \propto M$. By taking in mind that velocity dispersion in galaxy is determined mainly by DM, one may expect the existence of some relation between $M_{\rm SBH}$ and σ_e. We find the form of this relation in the following way. The condition for galaxy formation from a density fluctuation δ is

$$\delta_c = \delta_{\rm eq}(M) \frac{g(z)(1+z_{\rm eq})}{g(z_{\rm eq})(1+z)}, \tag{27}$$

where the function $g(z)$ is from (17), $\delta_c = 1.686$ is the threshold value of fluctuation for spherical collapse and M is a mass of the galaxy. For fluctuation amplitude $\delta_{\rm eq}(M)$ we take the r.m.s. fluctuation (10), and so we neglect the distributions but take into account only the mean values. Equation (27) gives implicitly the functional dependance $z(M)$. The density of a virialized object in $\varkappa = 18\pi^2$ times greater than the mean cosmological density $\rho_m(z) = \rho_{c,0}\Omega_{m,0}(1+z)^3$ of DM at the time corresponding to redshift z. This provides us with the relations between the radius, velocity dispersion in formed galaxy and galactic mass:

$$r(M,z) = \left[\frac{3M}{4\pi\varkappa\rho_m(z)}\right]^{1/3}, \quad \sigma_e(M) = \left[\frac{GM}{r(M,z(M))}\right]^{1/2}, \tag{28}$$

where $z(M)$ is derived from (27). By inverting the function $\sigma_e(M)$ and by using DM fluctuation spectrum (9) we find numerically

$$M \simeq 7\, 10^{11} \left(\frac{\sigma_e}{200 \text{ km s}^{-1}}\right)^{4.3} \text{M}_\odot. \tag{29}$$

If merging of primordial BHs into the one supermassive central BH proceeds effectively, from this relation we find a resulting mass of the central supermassive BH:

$$M_{\rm BH} = \psi\Omega_h M = 1.4\, 10^8 \left(\frac{\psi\Omega_h}{2\, 10^{-4}}\right) \left(\frac{\sigma_e}{200 \text{ km s}^{-1}}\right)^{4.3}, \tag{30}$$

where a factor ψ is related with a possible additional growth of the central BH by accretion of DM and baryonic matter. This model is in a reasonable agreement with observation data (26). The derived power index α in relation $M_{\rm SBH} \propto \sigma_e^\alpha$ is closer to one obtained in [48]. This power index in our model is completely defined by fluctuation spectrum at the galactic scales or, more definitely, by the value $n \simeq -2$ of power index. A possible dependence of an additional factor ψ on the mass, M, could modify a functional relation $M_{\rm SBH} = M_{\rm SBH}(\sigma_e)$. Nevertheless, a simple case $\psi = const$ provides a good agreement of the derived $M_{\rm SBH}(\sigma_e)$ relation with observations (26). This relation is naturally realized in the model without accretion $\psi = 1$. We expect that a minor influence of accretion or universal accretion fraction $\psi = const$ in the resulting mass of the central supermassive BHs in the galactic centers may be explained in detailed gas dynamics models of galactic nuclei.

It must be noted that $M_{\rm SBH} - \sigma_e$ correlation is a general feature of stochastic mechanism of supermassive BHs formation and is revealed also in other models of primordial BHs formation, e. g. [29].

VII. DISCUSSION

We describe here a new model of protogalaxy formation with the cluster of primordial BHs as a source of initial density perturbation. The used mechanism of primordial BH formation [31, 36] provide us with a set of primordial

BH clusters of different total mass. This variety of initial conditions leads, therefore, to the variety of protogalaxies from the very beginning of their formation. In this paper we choose for numerical modeling only those BH clusters which produce the large number of small relatively protogalaxies. This model predicts the very early galaxy and quasar formation. An other inevitable consequence of this model it the existence of intermediate mass BHs beyond the dynamical centers of galaxies and in the intergalactic medium. May be one of these type intermediate mass BHs was already observed by the X-ray Chandra telescope in the galaxy M82 [49].

More definitely in this model the protogalaxies are formed at redshift $z = 15$. These induced protogalaxies have initially the following parameters: a constituent total mass of DM $M_{\rm DM} = 2.2\,10^8 {\rm M}_\odot$, a virial radius 1.8 kpc, a mass of central BH $M_{\rm BH} = 7.2\,10^4 M_\odot$. In the following cosmological and dynamical evolution, these protogalaxies are assembled by hierarchical clustering into the nowadays galaxies. The clustering process occurs in a stochastic manner and leads to the specific correlation between the central supermassive BH mass and galactic bulge velocity dispersion [29]. An alternative proposed scenario is based on the initial large primordial BH clusters, when a resulting galaxy contains a single primordial BH growing due to accretion of ambient gas and stars and producing early quasar activity [32].

It is worth to estimate in the framework of our model a probability to find a nowadays galaxy without supermassive BH. Induced galaxies (with a central cluster of primordial BHs) and ordinary small protogalaxies have mass $M_{\rm DM} = 10^8 M_\odot$, while the modern galaxies are much more massive, $M_{\rm DM} = 10^{12} {\rm M}_\odot$. The merging of induced galaxy with an ordinary protogalaxy produces a next generation protogalaxy with the massive central BH. Therefore, about 10^4 collisions is required to form the nowadays galaxy. Suppose that an amount of induced galaxies is about 0.1% comparing with the ordinary ones. A corresponding probability to find a modern galaxy without supermassive BH is less than $0.999^{10000} \simeq 4.5\,10^{-5}$. Hence, even a very small fraction of induced galaxies is able to explain the observable abundance of AGN.

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